Adaptive controller design based on relocation of sample zeros for digital control systems

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Abstract: Adaptive control requires stability of the zero of objective systems to be cancelled by the controller. However the sample zeros generated by discretization of continuous-time systems are often unstable in many cases. This fact has discouraged application of adaptive controllers that rely on pole-zero cancellation to continuous-time systems. In this paper, we utilize Taylor series of the sample zeros to relocate the sample zeros toward the stable region and propose an approach to design adaptive controllers.

Keywords: Adaptive control, digital control, sample zero

1. INTRODUCTION

Various types of adaptive control have been developed for continuous and discrete-time systems for a half century. Since most of the adaptive controller is realized as computer programs, discretization by sample and hold operations is indispensable when the adaptive control is applied to continuous-time systems. In most cases, however, the discretization leads to unstable zeros of the transfer functions, which hamper the stable pole-zero cancellation that is one of the prerequisites for application of adaptive controllers. For example, let's consider the continuous-time transfer function of a DC motor

$$G(s) = \frac{65.3}{s(0.123s+1)} \tag{1}$$

with sampler and zero-order hold (ZOH) of the time period $\tau = 0.01$. Then we have the discrete-time transfer function

$$H(z) = \frac{B(z)}{A(z)} = \frac{0.0258(z+0.973)}{(z-1)(z-0.921)} \tag{2}$$

which has a zero near -1. Introducing a controller which has the pole to cancel the zero leads to long-lasting vibration and instability of the control system (e.g. Fig. 1). This hampers application of the majority of adaptive control to the DC motor. Moreover it has been well-known that such a unstable zero is very common for continuous-time systems discretized by sample and hold operations[1]. This fact has discouraged application of

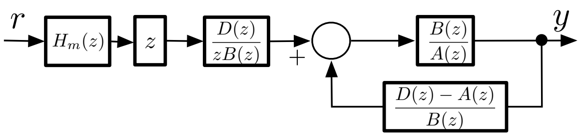

Fig. 1 Model following control.

the digital controller that relies on pole-zero cancellation to continuous-time systems.

In recent years, however, the authors proposed an approach to design post-filter for ZOH to relocate the zeros generated by discretization[4]. The post-filter was successfully applied to DC motor with digital model following control systems based on the pole-zero cancellation. In this paper, we apply this approach to design of digital adaptive control for continuous-time systems. Experimental results of model reference adaptive control applied to a DC motor are reported.

† Takuya Sogo is the presenter of this paper.

2. PROPERTIES OF TAYLOR EXPANSION OF THE PULSE TRANSFER FUNCTION AND SAMPLE ZERO

Consider a linear SISO time-invariant system (A_c, B_c, C) and let the transfer function be

$$\begin{aligned} G(s) = C(sI-A_c)^{-1}B_c &= \frac{\begin{vmatrix} A_c - sI & B_c \\ C & 0 \end{vmatrix}}{|A_c - sI|} \\ &= \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \\ &= \frac{K(s-q_1)\cdots(s-q_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}. \end{aligned} \tag{3}$$

Discretizing with a sampler and ZOH of the time period τ, we have the sampled-data system (A, B, C) where

$$A = \exp(A_c\tau) = \sum_{k=0}^{\infty} A_c^k \frac{\tau^k}{k!} \tag{4}$$

$$B = \int_0^{\tau} \exp(A_c t)dt B_c = \sum_{k=0}^{\infty} A_c^k B_c \frac{\tau^{k+1}}{(k+1)!} \tag{5}$$

The pulse transfer function is written as

$$H(z) = \frac{\begin{vmatrix} A - zI & B \\ C & 0 \end{vmatrix}}{|A - zI|} \tag{6}$$

$$= \frac{\beta_1 z^{n-1} + \beta_2 z^{n-2} + \cdots + \beta_n}{z^n + \alpha_1 z^{n-1} + \cdots + \alpha_{n-1} z + \alpha_n} \tag{7}$$

$$= \frac{C_\tau\{z-\gamma_1(\tau)\}\cdots\{z-\gamma_{n-1}(\tau)\}}{(z-e^{p_1\tau})(z-e^{p_2\tau})\cdots(z-e^{p_n\tau})} \tag{8}$$

In contrast with the simple relation between the poles of $G(s)$ and $H(z)$ as p_k and $e^{p_k\tau}(k=1,2,\cdots,n)$, there is no simple expression of the sample zeros $\gamma_k(\tau)$ and even the number of the sample zeros has no relation with the zeros of the continuous-time systems. From the expressions (4) , (5) and (6) , it is easily shown that the coefficients β_i $(i=1,\cdots,n)$ of $H(z)$ are expressed as

$$\beta_i=\sum_{k=0}^{\infty}\sum_{j=0}^{m} f_{k,j}(a_1,\cdots,a_n)b_j\tau^k \tag{9}$$

where $f_{k,j}$'s are multi-variable polynomials of $(a_1,\cdots,a_n)$, namely

$$f_{k,j}(a_1,\cdots,a_n)=\sum_l c_l a_1^{l_1}\cdots a_n^{l_n} \tag{10}$$

where c_l's are coefficient constants and $l_1,\cdots,l_n$ are non-negative exponential values. This implies that the sample zeros have a complex relationship with both the continuous-time zeros and poles.

However, it was shown[1] that the lower-order terms of the coefficient β_i with respect to τ vanish, i.e.

$$f_{k,j}(a_1,\cdots,a_n)=0 \tag{11}$$

for $k=0,\cdots,n-m-1$ and any j and the term for $k=n-m$ is a constant proportional only to b_0, i.e.

$$f_{n-m,0}(a_1,\cdots,a_n)=\text{constant} \tag{12}$$

$$f_{n-m,j}(a_1,\cdots,a_n)=0 \ \text{ for } j=1,\cdots,m \tag{13}$$

Moreover, it is conjectured by the authors that Taylor expansions of the coefficients β_i have generalized simple properties about the multi-variable polynomials [5]:

Conjecture 1: *The summation in*

$$f_{k,j}(a_1,\cdots,a_n)=\sum_l c_l a_1^{l_1}\cdots a_n^{l_n} \tag{14}$$

consists only of the terms $c_l a_1^{l_1}\cdots a_n^{l_n}$ that satisfy

$$1\cdot l_1+2\cdot l_2\cdots+n\cdot l_n+j+n-m\le k \tag{15}$$

This implies that the lower terms with respect to τ of the coefficients β_i of the pulse transfer function depend only on the coefficients a_i and b_j with smaller indices of the continuous-time transfer function.

For example, let's consider the case of $(n,m)=(3,1)$. Then we can see the coefficients of the pulse transfer function have only terms of τ^2 and higher with the coefficients according to the inequality (15):

$$\begin{aligned}\beta_1=&b_0\frac{\tau^2}{2}+(-a_1b_0+b_1)\frac{\tau^3}{3!}\\&-(a_2b_0+a_1b_1)\frac{\tau^4}{4!}\cdots\end{aligned} \tag{16}$$

$$\begin{aligned}\beta_2=&0\cdot\frac{\tau^2}{2}+(-a_1b_0+4b_1)\frac{\tau^3}{3!}\\&+(2a_1^2b_0-8a_1b_1)\frac{\tau^4}{4!}\cdots\end{aligned} \tag{17}$$

$$\begin{aligned}\beta_3=&-b_0\frac{\tau^2}{2}+(2a_1b_0+b_1)\frac{\tau^3}{3!}\\&+\{(-3a_1^2+a_2)b_0-3a_1b_1\}\frac{\tau^4}{4!}\cdots\end{aligned} \tag{18}$$

The coefficients of τ^3 and τ^4 depend only on a_1 and (a_1,a_2), respectively, among the coefficients (a_1,a_2,a_3) of the denominator of the continuous-time transfer function. These expressions of the coefficients lead to simple Taylor expansions of two sample zeros[3, 5].:

$$\begin{aligned}\gamma_1(\tau)=&-1+\frac{a_1-b_1/b_0}{3}\tau\\&-\frac{(a_1-b_1/b_0)^2}{3^2}\frac{\tau^2}{2!}+\cdots\end{aligned} \tag{19}$$

$$\gamma_2(\tau)=1-b_1/b_0\tau+\frac{(b_1/b_0)^2}{2!}\tau^2+\cdots \tag{20}$$

3. RELOCATION OF THE SAMPLE ZERO BY POST-FILTER TO ZOH

We consider the continuous-time system (3) with $(n,m)=(2,0)$ and introduce a filter

$$C(s)=\frac{s-q}{s-p} \tag{21}$$

between ZOH and the continuous-time system $G(s)$ (Fig. 2). It should be noted that $C(s)G(s)$ with $(n,m)=(2,0)$ is equivalent to $G(s)$ with $(n,m)=(3,1)$ and therefore the sample zeros are expressed as (19) and (20). Since $a_1=-(p_1+p_2+p)$ and $b_1/b_0=-q$, it is possible to adjust approximately the location of the sample zero by the values of (q,p) unless each coefficients of (19) and (20) is much bigger than τ^{-k}. When the sample period τ

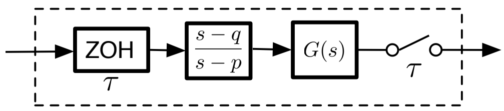

Fig. 2 Zero relocation filter

is sufficiently smaller than the poles p_1 and p_2, we truncate the terms of the Taylor series more than 2nd order of the sample zero and approximately minimize the absolute value of the sample zeros $|\gamma_k(\tau)|$ in order to maximize the convergence rate. Since those approximation expressions are 2nd order polynomials of the free parameters (q,p), the minimizers are easily found as

$$q=-1/\tau,\quad p=-4/\tau-(p_1+p_2) \tag{22}$$

For the DC motor (1) with $\tau=0.01$, we choose the filter as

$$C(s)=\frac{s+100}{s+391.9} \tag{23}$$

The discrete-time transfer function of the DC motor is transformed by the filter into

$$H(z)=\frac{0.013913(z+0.4505)(z-0.3681)}{(z-1)(z-0.9219)(z-0.01987)} \tag{24}$$

zeros of which can be canceled by the faster decaying poles. Here it should be noted that the parameters of the filter $C(s)$ are dominated by $1/\tau$ and not very sensitive to the poles p_1 and p_2 because the sample period τ is sufficiently smaller than the poles p_1 and p_2. This implies that the function of the zero relocation filter is robust against variation of the parameters of $G(s)$. For example, we assume that the time constant $T = 0.123$ of (1) varies to be 1.23 and we still apply the same filter as (23) to the varied one. Nonetheless we have the discrete-time system as

$$H(z) = \frac{0.0014297(z + 0.464)(z - 0.3682)}{(z - 1)(z - 0.9919)(z - 0.01987)} \quad (25)$$

zeros of which still can be canceled by stable poles. Based on this fact, we design a model reference adaptive controller with the fixed filter for zero relocation.

4. ADAPTIVE CONTROLLER DESIGN

Thanks to the successful relocation of the sample zeros, we can design a stable model following controller that includes the poles to cancel the sample zeros (Fig.1). Here we design a recursive least square (RLS) estimator[2] for the coefficients of the numerator and denominator of $H(z)$ and utilize the estimated coefficients to adjust the coefficients of the controllers (Fig. 3).

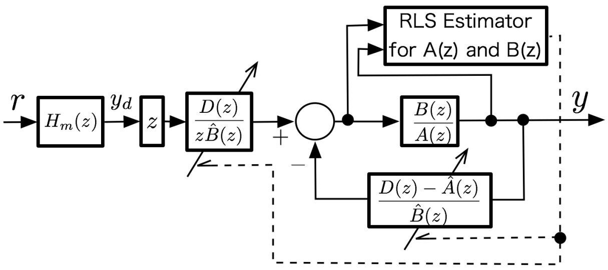

Fig. 3 Adaptive control based on RLS estimator

5. NUMERICAL SIMULATION

The RLS-based adaptive controller is applied to the DC motor model (1) whose parameter varies, i.e.

$$G(s) = \frac{K}{s(Ts + 1)} \quad (26)$$

where the time constant T changes from $T = 0.123$ to $T = 1.23$ at the time $t = 10$[s]; K is fixed as 65.3. As shown in Section 3, the relocation filter (23) works well for both values of the parameter T. The model following controller with

$$D(z) = (z - \pi_1)\cdots(z - \pi_n), \quad (27)$$

$\hat{B}(z)$ and $\hat{A}(z)$ adjusted on-line by the RLS estimator is appied to the above mentioned model with the filter and numerical simulation is conducted. We set the desired output trajectory $y_d(t)$ as the response of

$$H_m(z) = \frac{(z - 0.01)^2}{(z - 0.9)^3} \quad (28)$$

to the unit pulse of the period 4[s] with the duty ratio 50%, i.e. $r \equiv 1$ for $4(k-1) \le t < 4k - 2$ and $r \equiv 0$ for

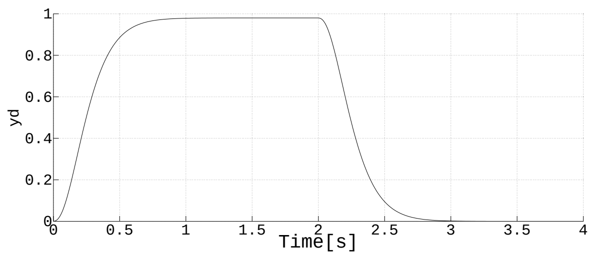

Fig. 4 Desired output trajectory y_d

$4k - 2 \le t < 4k$ where $k = 1, 2, \cdots$. (see Fig. 4) When the poles π_k ($k = 1, 2, 3$) of the feedback block is chosen as either $\pi_1 = \pi_2 = \pi_3 = 0.8$ or $\pi_1 = \pi_2 = \pi_3 = 0.01$, the output y stably tracks the desired trajectory even after the change of the parameter. Fig. 5∼ 8 show the simulation results for the latter case; Fig. 5 shows that the output deviation $y(k\tau) - y_d(k\tau)$ vanishes shortly after the simulation begins and the deviation stays attenuated even after the parameter varies. Fig. 5 shows that the

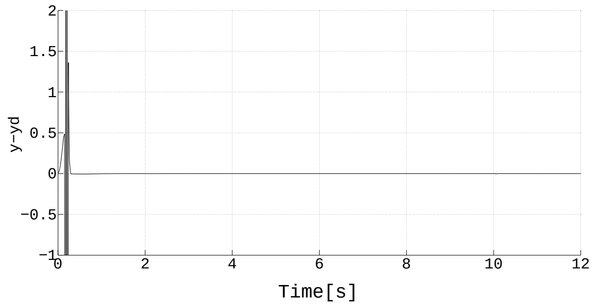

Fig. 5 Output deviation $y - y_d$

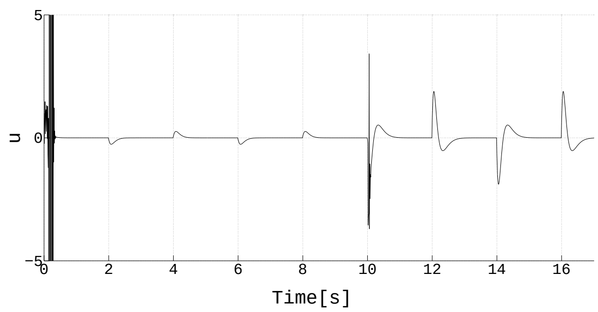

Fig. 6 Input u

input generated by the controller whose poles cancels the zeros is free from vibration thanks to the relocation of the zeros. Fig. 7 and 8 show the convergence of the estimated parameters $(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)$ and $(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$ of

$$\hat{H}(z) = \frac{\hat{B}(z)}{\hat{A}(z)} = \frac{\hat{\beta}_1 z^2 + \hat{\beta}_2 z + \hat{\beta}_3}{z^3 + \hat{\alpha}_1 z^2 + \hat{\alpha}_2 z + \hat{\alpha}_3} \quad (29)$$

to the ones of $H(z)$.

To demonstrate the efficacy of the relocation of the zeros, numerical simulation for the same DC motor model

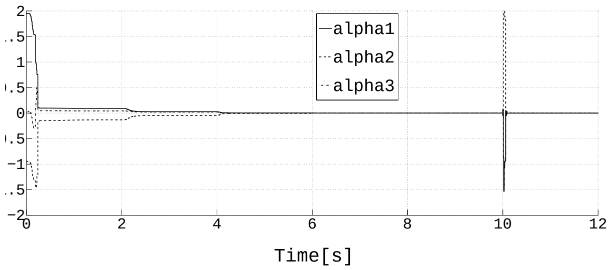

Fig. 7 Estimation deviations: $(\hat{\alpha}_1-\alpha_1, \hat{\alpha}_2-\alpha_2, \hat{\alpha}_3-\alpha_3)$

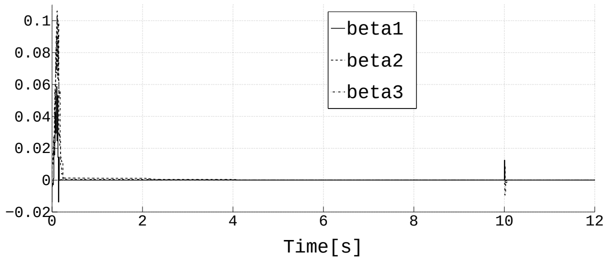

Fig. 8 Estimation deviations: $(\hat{\beta}_1-\beta_1, \hat{\beta}_2-\beta_2, \hat{\beta}_3-\beta_3)$

without the zero relocation filter is conducted. Application of the RLS-based adaptive controller to the model without the zero relocation filter results in vibration and divergence of the input for the poles $\pi_1 = \pi_2 = 0.8$ after the change of the parameter and for the poles $\pi_1 = \pi_2 = 0.01$ shortly after the simulation begins (Fig. 9); neither tracking to the desired trajectory nor parameter estimation can be achieved. Even when the controller parameters are fixed and set as the true values to avoid the divergence of the adaptive control, the unstable vibration is observed for the input signal(Fig. 10).

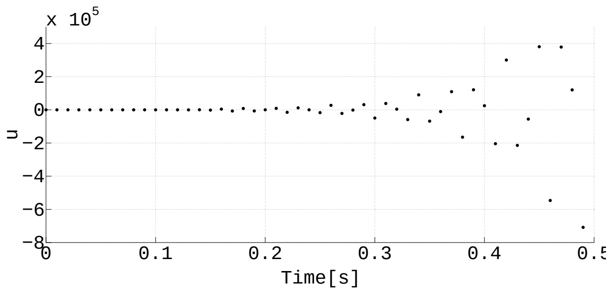

Fig. 9 Input u by the adaptive controller for the model w/o the relocation filter

6. CONCLUDING REMARKS

In this paper, we propose an approach to relocate unstable sample zeros based on the simple algebraic properties of Taylor expansion of the pulse transfer function coefficients. It is demonstrated that the approach is successfully applied to designing stable adaptive controller for digital control systems by introducing a post-filter of ZOH to relocate the unstable sample zeros. Simulation results demonstrate the efficacy of the proposed adaptive controller.

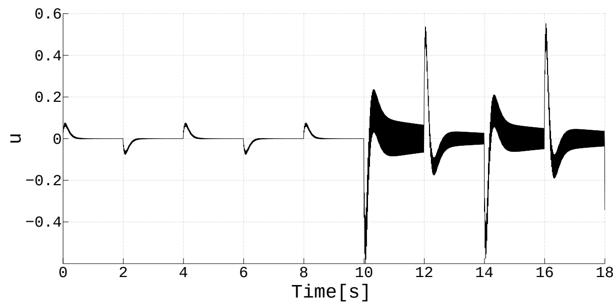

Fig. 10 Input u by the non-adaptive controller for the model w/o the relocation filter

In this paper, we applied the simple properties of the pulse transfer function only to relocation of sample zeros of DC motor models. However, the obtained Taylor expansion of the coefficients (16), (17) and (18) imply application possibility of them to efficient estimation of the parameters $(\beta_1, \beta_2, \beta_3)$ because they are approximately constrained by simple polynomials of the continuous-time transfer function coefficients (b_0, b_1) and (a_1, a_2, a_3). Those approaches are currently under consideration as future work.

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